International Journal of Scientific Research in Engineering and Management (IJSREM)
Volume: 04 Issue: 09 | Sept -2020 ISSN: 2582-3930

Fuzzy e-Epidemic Model on Covid-19: An Overview

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Abstract

This paper has an aim to dynamically formulate an e-epidemic compartmental model using fuzzy mathematics in human population. The most recent edition of individual corona virus said to be COVID-19 came out as a sudden pandemic disease within human population and in the absence of vaccination and proper treatment till date, it daunting threats heavily to human lives, infecting more than 23.9 M people and death more than 820 K people in more than 208 countries across the globe as on August 25, 2020, which is highly alarming. When no treatment or vaccine is available till date and to avoid COVID-19 to be transmitted in the community, to follow the minimum criteria (social distancing, using masks etc.) is the only way to prevent the disease. We have studied the ways of classical basic reproduction number as well as fuzzy basic reproduction number means at what time both differ and when matched. We have also analyzed the control strategies. For this, we have taken three cases for disease in population as: *low*, *medium* and *high* which tells that at what condition the disease will invade the population.

Key Words: Epidemic Model; Diseases; Covid-19; Fuzzy Sets/Logic; Population; Fuzzy basic reproductive number;

1. INTRODUCTION

As per the report of World Health Organization (WHO) over 208 countries across the globe with total confirmed cases more than 23.9 M people and death more than 820 K people due to corona virus as on August 25, 2020, not only are people widely calling Covid-19 both an epidemic and pandemic, but they are also calling it an outbreak. An outbreak is a sudden breaking out or occurrence or eruption. When referring to a transferable infection, an outbreak is specifically a sudden rise in cases, especially when it is only or so far affecting a relatively localized area. A pandemic disease is an epidemic that has spread over a large area, that is, it's prevalent throughout an entire country, continent, or the whole world. A pandemic is defined as an epidemic occurring worldwide, or over a very wide area, crossing international boundaries and usually affecting a large number of people [1]. The majority of the people contaminated with the COVID-19 virus experiences mild to modest respiratory sickness and make progress with no requiring special treatment. Older people and the individuals with some medical problems like cardiovascular disease, diabetes, chronic respiratory disease, and cancer are more to be expected to increase serious disease. The COVID-19 virus spreads primarily through droplets of saliva or discharge from the nose when an infected person coughs or sneezes [2, 3].

In the epidemic mathematical models, the estimation of parameters is usually based on statistical methods, starting from data obtained experimentally to the choice of the method adapted to their identification. In this paper, we have used the concept of Fuzzy Set/Logic Theory which is an extension of the concept of a crisp set. It also deals with the techniques of computing and manipulating with fuzzy sets. Though Fuzzy epidemic models for human infectious disease have been well studied [4 – 13] but very few applications and research papers, using fuzzy logic, in the transmission of malicious objects in computer network exists in literature [14, 15].

2. THE SIMPLE CLASSICAL SS_EQ_IRS MODEL

A classical e-epidemic SS_eQ_iRS model demonstrates the dynamics of propagation of covid-19 amongst several classes of people. We have also supposed that, there is neither vital dynamics (i.e. the rates of birth and mortality (reason other than attack of covid-19) are not considered), nor additional disease fatality rate in the population. The flow of covid-19 in population can be represented as (figure 1):

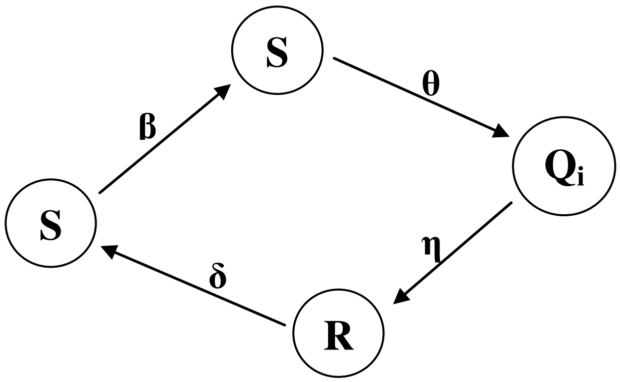

Figure 1: Schematic diagram for flow of covid-19 in population

The system of ordinary differential equations of this model is given by:

$$\frac{dS}{dt} = -\beta S Q_i + \delta R$$

$$\frac{dS_e}{dt} = \beta S Q_i - \theta S_e - d S_e \qquad (1)$$

$$\frac{dQ_i}{dt} = b - \theta S_e - \eta Q_i - d Q_i$$

$$\frac{dR}{dt} = \eta Q_i - \delta R$$

where, $S + S_e + Q_i + R = 1$, (2)

and nomenclature for the model is as follows:

S : the class of proportional susceptible population

S_e : the class of proportional population that don't meet the minimum criteria to prevent COVID-19 (i.e. mask, social distancing etc.)

Q_i : the class of proportional infected population that are quarantined (may be home, hospitals etc.)

R : the class of proportional recovered population

β is the rate of contact; θ is the rate of infection, η is the recovery rate and δ is the rate of susceptible after recovery. We now assume an expansion of the SS_eQ_iRS dynamic model incorporating heterogeneities, taking into account that nodes with different amount of covid-19s contribute differently to the covid-19 transmission.

3. THE E-EPIDEMIC SS_EQ_IR FUZZY MODEL

To change a simple SS_eQ_iRS model into SS_eQ_iRS fuzzy model, we assume that the inhabitants' heterogeneity is given by the covid-19 load of infected population. It means to say that, the higher the covid-19 load, the higher will be the chance of covid-19 transmission. So, we suppose $\beta = \beta(x)$ is the chance of covid-19 spread between S and S_e classes with a sum of covid-19 *x*. Now, it may probable that, some values of β are more feasible than others and that turns β into a membership function of a fuzzy number. So, to obtain the membership function β, we assume that when the sum of covid-19 in a population is comparatively low, the chance of covid-19 spread is very minor and then there is a least amount of covid-19 x_{min} for the propagation. Now, for some sum of covid-19 x_M, the chance of spreading is maximum and equal to one. However, we assume that the amount of covid-19 in a node is always limited by x_{max}. Hence, we define the membership function (depicted in figure 2) as,

$$\beta(x) = \begin{cases} 0, & \text{if } x \le x_{min} \\ \dfrac{x - x_{\min}}{x_M - x_{\min}}, & \text{if } x_{min} < x < x_M \\ 1, & \text{if } x_M \le x < x_{max} \end{cases} \qquad (3)$$

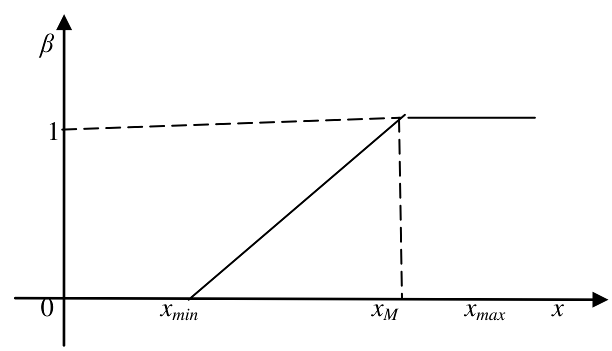

Figure 2: Fuzzy coefficient of covid-19 transmission $\beta = \beta(x)$

Now, $\theta = \theta(x)$ is the chance of transmission to occur in between S_e and Q_i which is also function of covid-19 load. The higher the infection in the network the higher it will be chance of quarantine, that is, θ is an increasing function of *x*, defined as (depicted in figure 3).

$$\theta(x) = \frac{1-\theta_0}{x_{\max}} x + \theta_0 \qquad (4)$$

where, $\theta_0 > 0$ is the lowest rate of infection.

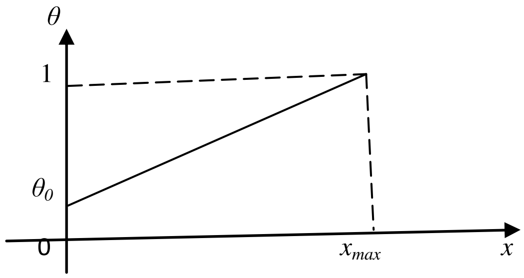

Figure 3: Fuzzy coefficient of infection $\theta = \theta\,(x)$

Also, the recovery rate $\eta = \eta\,(x)$ of nodes is also a function of covid-19 load. The higher the covid-19 load, the longer it will take to recover from infection. That is, η should be a decreasing function of x which is represented in figure 4.

$$\eta\,(x) = \frac{\eta_0 - 1}{x_{\max}}\,x + 1 \qquad (5)$$

where, $\eta_0 > 0$ is the lowest recovery rate.

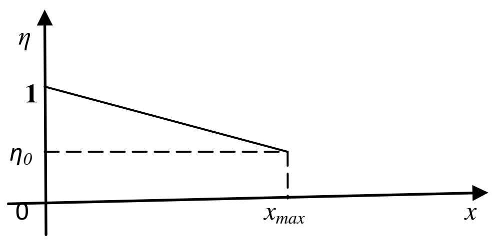

Figure 4: Recovery fuzzy rate $\eta = \eta\,(x)$

Next, it is possible that the recovered individual may be susceptible again. Then, we define $\delta = \delta\,(x)$, the rate of susceptible after recovery. The higher we use the mask, sanitization, social distancing etc., the higher it will be susceptible after recovery. So, it should be a increasing function of x depicted in figure 5.

$$\delta\,(x) = \frac{1 - \delta_0}{x_{\max}}\,x + \delta_0 \qquad (6)$$

where, $\delta_0 > 0$ is the lowest susceptibility rate after recovery.

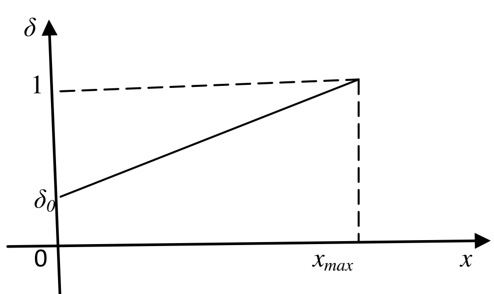

Figure 5: Fuzzy rate of susceptibility after recovery $\delta = \delta\,(x)$

Now, we have also assumed that the amount of covid-19s is different for different individual of the network. It means that x be a fuzzy number with a triangular shape according to the following membership function (depicted in figure 6).

$$\sigma\,(x) = \begin{cases} 1 - \dfrac{|x - \overline{x}|}{\varepsilon}, & \text{if } x \in [\overline{x} - \varepsilon,\ \overline{x} + \varepsilon] \\ 0, & \text{if } x \notin [\overline{x} - \varepsilon,\ \overline{x} + \varepsilon] \end{cases} \qquad (7)$$

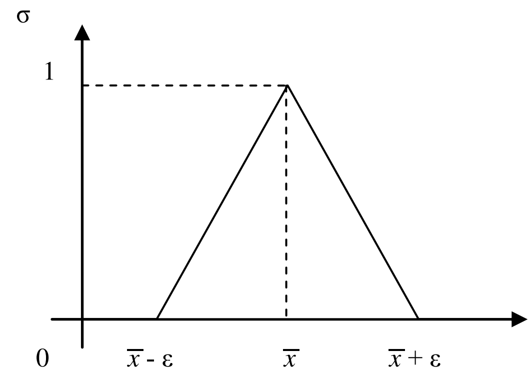

Figure 6: Membership function of the variable x, amount of the covid-19s - σ [39]

where, the parameter $\overline{x}$ is a central value and ε provides the scattering of each one of the fuzzy sets assumed by x. For a fixed $\overline{x}$, $\sigma\,(x)$ can have a linguistic sense, such as *low*, *medium*, *high* and so on.

4. SOLUTIONS AND EQUILIBRIUM POINTS

In order to know about the evolution of infected population, that is, the number of infected people increases indefinitely or not, we study the stability of equilibrium points. Then from the system (1) and equation (2), we have,

$$\frac{dS}{dt} = -\beta S Q_i + \delta S$$

$$\frac{dS_e}{dt} = \beta S Q_i - \theta S_e \quad \frac{dQ_i}{dt} = \theta S_e - \eta(1 - S - S_e - Q_i)$$

Then, for the equilibrium points, we take, $\frac{dS}{dt} = 0 = \frac{dS_e}{dt} = \frac{dQ_i}{dt}$, and then we get four equilibrium points P_1 (1, 0, 0, 0), P_2 (0, 1, 0, 0), P_3 (0, 0, 1, 0) and P_4 ($\frac{\theta}{\beta}, \frac{\eta\beta - \theta\delta - \eta\delta - \eta\theta}{\eta\beta}, \frac{\delta}{\beta}, \frac{\theta\delta}{\eta\beta}$) for the system (1). The analysis of the stability of the system (1) indicates that the points P_1, P_2, P_3 are unstable while the point P_4 (with $\delta \le \beta$) is asymptotically stable which shows that the number of infected population will stabilize in $\frac{\theta\delta}{\eta\beta}$ even if the number increases. However, $\frac{\theta}{\beta}$ and $\frac{\eta\beta-\theta\delta-\eta\delta-\eta\theta}{\eta\beta}$ of the population will not be affected. Now, by using fuzziness, means that, taking into account the covid-19 load, we have,

$$P_4\left(\frac{\theta(x)}{\beta(x)}, \frac{\eta(x)\beta(x)-\theta(x)\delta(x)-\eta(x)\delta(x)-\eta(x)\theta(x)}{\eta(x)\beta(x)}, \frac{\delta(x)}{\beta(x)}, \frac{\theta(x)\delta(x)}{\eta(x)\beta(x)}\right)$$

Since, $\frac{\delta(x)}{\beta(x)} < 1$ and P_4 is asymptotically stable, then for a value of bifurcation for x is x^*, the solution of the equation $\delta(x) = \beta(x)$ will be,

$$x^* = \frac{x_M x_{\max}}{x_{\max} + (1-\delta_0)(x_M - x_{\min})} \tag{8}$$

and $x_{min} \le x^* \le x_M$

The value of the bifurcation of the model is the covid-19 load x^*, because the model has three unstable equilibrium points, if $x < x^*$ and the model has only one asymptotically stable point, if $x > x^*$. Here we mean to say that, we consider x^* as a parameter related to covid-19 control and if a covid-19 is installed in a network then it guarantees that, the covid-19 load x is not greater than x^*.

5. THE BASIC REPRODUCTION NUMBER (R_0)

As we know that, in classical model, the basic reproduction number can be found all the way through the limited study of the stability of the trivial equilibrium point. That is, $R_0 = \frac{\beta}{\theta}$, which tells that, the infection will not exist in the population if $\frac{\beta}{\theta} < 1$ and it will attack the network if $\frac{\beta}{\theta} > 1$. Now, due to fuzziness, since, $\beta = \beta(x)$ and $\theta = \theta(x)$, then we can write, $R_0 = \frac{\beta(x)}{\theta(x)}$. By this discussion, in order to control the disease transmission, we can impose $max\{R_0(x)\} < 1$. However this can be acute approach. Therefore, possibly, it is better to assume an average value of $R_0(x)$. For this and for a given triangular fuzzy number $\sigma(x)$, we define the fuzzy basic reproduction number by,

$$R_0^f = \frac{FEV[\theta_0 R_0(x)]}{\theta_0} \tag{9}$$

Here, $R_0(x)$ can be greater than one but $\theta_0 R_0(x) \le 1$, so that, R_0^f is well-defined. Here, R_0^f can be taken as the average number of lesser cases of infected individuals established into a susceptible class. So, to define $FEV[\theta_0 R_0(x)]$, we take a fuzzy measure μ by using possibility measure as,

$$\mu(A) = \sup_{x\in A} \sigma(x) \ ; \quad A \subset R$$

It means that the infectivity of a group is the one presented by the individual in population belonging to the group with maximal infectivity. Now, in the case of R_0^f we suppose that the sum of covid-19 x in the population has a linguistic meaning classified as *low*, *medium* and *high*. Then the fuzzy sets given by the membership function $\sigma(x)$ for the three different cases:

(*i*) *low*, if $\bar{x} + \varepsilon < x_{min}$;

(*ii*) *medium*, if $\bar{x} - \varepsilon > x_{min}$ and $\bar{x} + \varepsilon \le x_M$ and

(*iii*) *high*, if $\bar{x} - \varepsilon > x_M$.

6. COMPARISON BETWEEN R_0 AND R_0^f

Here, we study the three cases, already discussed in earlier part, for the sum of covid-19-load. Then for any of the three cases, we have,

$$\frac{\beta(\bar{x})}{\theta(\bar{x})} < \frac{FEV[\theta_0 R_0(x)]}{\theta_0} < \frac{\beta(\bar{x}+\varepsilon)}{\theta(\bar{x}+\varepsilon)}$$

That is, $R_0(\bar{x}) < R_0^f < R_0(\bar{x}+\varepsilon)$

Now, since $R_0(x) = \frac{\beta(x)}{\theta(x)}$ is curved and continuous, then by intermediate mean value theorem there exists only one x', with $\bar{x} < x' < \bar{x}+\varepsilon$, such that, $R_0^f = R_0(x') > R_0(\bar{x})$ (10)

It means to say that, there is an amount of covid-19s x' where R_0 (classical) and R_0^f (fuzzy) coincide. Also, the average value of the number of secondary cases (R_0^f) is higher than the number of secondary cases due to the average sum of covid-19s $R_0(\bar{x})$.

7. COVID-19 CONTROL STRATEGIES

Based on the above analysis, we say that the system of equations (1), as a family of systems depending on the parameter x and there is one which plays an important role to discuss about the covid-19 control. According to (10), $R_0(\bar{x})$, which plays a vital role, as an indicator of covid-19 control, forces us to evaluate the correct parameter for the population, that is $R_0(x')$. To describe the dynamics of covid-19s in the network, we will study the covid-19 control in the population by taking $R_0(x') = R_0^f$:

(*i*) for the amount of covid-19 load, *low*, we have, $x' < \bar{x} + \varepsilon \geq x_{min}$. So, $R_0(x') = 0$ and the covid-19 will not establish itself (of course, with the preventive measures for Covid-19 taken into account).

(*ii*) for the amount of covid-19 load, *high*, we have, $x' > \bar{x} > \bar{x} + \varepsilon \geq x_M$. So, $R_0(x') = \frac{1}{\theta(\bar{x})} > 1$ indicates that the covid-19 will attack in the network.

(*iii*) in case of medium amount of covid-19 load, (*a*) if $x^* > x'$ then $R_0(x') = \frac{\beta(x')}{\theta(x')} < \frac{\beta(x^*)}{\theta(x^*)} = 1$ which means that the disease will not attack in the network and (*b*) if $x^* < x'$ then

$$R_0(x') = \frac{\beta(x')}{\theta(x')} > \frac{\beta(x^*)}{\theta(x^*)} = 1$$ which indicates that the disease will not attack in the network. But, in case of Covid-19, it will establish only if an individual maintains the minimum preventive measures (social distancing etc.) as the attacking behavior of Covid-19 changes among individuals because immunity system matters a lot in this case.

8. CONCLUSION

An e-epidemic SS_eQ_iRS model has been developed for the transmission of disease using fuzzy sets/logic in the population. We have discussed the stability of the system as well as the behavior of different classes with respect to time. In this model we have taken the parameters, used to develop the system of differential equations, as a membership function of x which is a fuzzy number and then we found the fuzzy basic reproduction number R_0^f, by the help of the classical basic reproduction number R_0 and observed that, there will be an amount of infected population where both coincide (by the help of Intermediate Value Theorem). We have also observed that the characteristics of disease, in the sense of R_0^f, the sum of covid-19s in the population has a linguistic meaning classified as: *low*, *medium* and *high*. In this way, we conclude that, the three cases of control strategies tell that when the amount of infection will be low, covid-19s will not be in the network, for the high amount of infection, covid-19 will invade and for the medium amount of infection, covid-19 may or may not be in the population with the preventive measures for Covid-19 taken into account. Till date when no vaccine or treatment is available for Covid-19, then it is well established that social distancing and hospital quarantine (for positive cases) is the only best treatment.

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